# A DESCRIPTOR SYSTEMS PACKAGE FOR MATHEMATICA

A.I. Vardulakis, N. P. Karampetakis, E. Antoniou, P. Tzekis and S. Vologiannidis

Department of Mathematics
Aristotle University of Thessaloniki
Thessaloniki 54006, Greece
http://anadrasis.math.auth.gr

# Outline of the presentation

- **Control System Professional**
- **Polynomial Control Systems**
- **Descriptor Control Systems**

# Mathematica and Control

## Control System Professional

- ***Control System Professional*** handles linear systems described by **state-space equations** and **proper transfer functions**.
  - Time-Domain Response Analysis
  - System Interconnections
  - Controllability and Observabillity
  - Realizations Construction and Conversion
  - Feedback Control Systems Design
  - Optimal Control Systems Design
  - Linearization tools

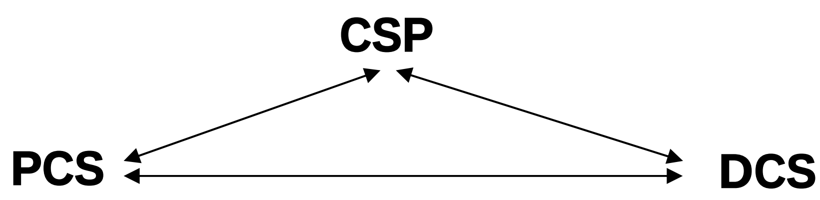
# Mathematica and Control

## Polynomial Control Systems

- Polynomial Control Systems developed by Prof. Munro handles the general class of **polynomial matrix descriptions (PMDs).**
  - Model transformations
  - System analysis
  - System design

# Objectives of the **descriptor systems package**

- Extend the functionality of the Control Systems Professional package in order to handle **descriptor state space representations** and **improper transfer functions**.
  - Manipulation of polynomial and rational matrices
  - Introduction of descriptor state space systems as data objects
  - Extension of the functions of CSP concerning
    - System analysis
    - Time-Domain Response Analysis
    - Synthesis and design techniques
- Maintain compatibility with the existing infrastructure of Control Systems Professional and Polynomial Control Systems.



# Manipulation of polynomial and rational matrices

- New functions for the study of **rings of rational functions with poles in a prescribed region of the complex plane** as well as for rational matrices with entries coming from these rings
  - the ring of rational functions with **no poles in the complex plane** (polynomials) *(ForbiddenPolesArea->FiniteComplex)*
  - the ring of rational functions with **no poles at infinity** (proper functions) *(ForbiddenPolesArea->InfinityPoint)*
  - the ring of rational functions with **no poles in the extended right half complex plane** (proper and Hurwitz stable rational functions) *(ForbiddenPolesArea->HurwitzStable)*
  - the ring of rational functions with **no poles outside the unit circle** (proper and Schur stable rational functions)

    *(ForbiddenPolesArea->SchurStable)*

# Manipulation of polynomial and rational matrices

## Problems studied over different rings

- Division between two rational functions
- Greatest common divisor and least common multiple
- Coprimeness
- Smith - McMillan form
- Solutions of rational matrix Diophantine equations

# Descriptor State Space Models

Consider a descriptor system described by a set of linear differential and/or algebraic equations of the form:

$$E\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

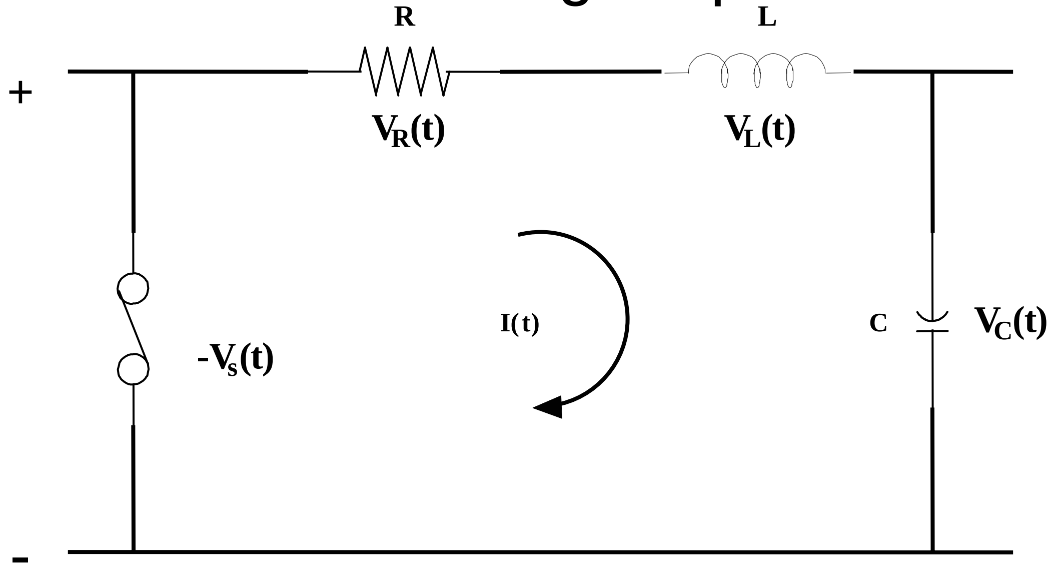
$where\ E, A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}, D \in R^{p\times m}$ and E possibly non-singular with $\det\left(sE - A\right) \neq 0$.

- A **Descriptor state space system data object** named "DescriptorStateSpace" has been added in Mathematica.
- **Transformations** between Transfer functions, Descriptor State Space and PMDs are available

# Descriptor State Space Models

## The descriptor state-space model of a simple RLC circuit.

- Consider the following simple RLC circuit (Dai 1989)




R, L, C stand for the resistor, inductor and capacity quantities respectively.
$V_S$ is the voltage source (control input), and $V_R$, $V_L$, $V_C$ are the corresponding voltages.

# Descriptor State Space Models

The descriptor state-space model of a simple RLC circuit.

Definition

```
e={{L,0,0,0},{0,0,1,0},{0,0,0,0},{0,0,0,0}};
a={{0,1,0,0},{1/C,0,0,0},{-R,0,0,1},{0,1,1,1}};
b={{0},{0},{0},{-1}};
c={{0,0,1,0}};
```

```
dss=DescriptorStateSpace[e, a, b, c]
```

$$\left(\begin{array}{cccc|cccc|c} L & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 \\ \hline & & & & 0 & 0 & 1 & 0 & 0 \end{array}\right)^{\mathcal{D}}.$$

E B A D C

```
TransferFunction[dss]
```

$$\left(\frac{1}{C s (R + L s) + 1}\right)^{\mathcal{T}}$$

# Descriptor State Space Models

The descriptor state-space model of a simple RLC circuit.

```
EquationForm[dss, StateVariables → {"I", V_L, V_C, V_R}, InputVariables → V_S]

WeierstrassCanonicalForm[dss /. R → 1 /. C → 1 /. L → 1]
```

$$\begin{pmatrix} L & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\mathrm{I}} \\ \dot{V}_L \\ \dot{V}_C \\ \dot{V}_R \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{C} & 0 & 0 & 0 \\ -R & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{I} \\ V_L \\ V_C \\ V_R \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} V_S$$

$$y = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{I} \\ V_L \\ V_C \\ V_R \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & -\frac{\sqrt{\frac{3}{95}}}{2} \\ 0 & 1 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & \frac{\sqrt{\frac{5}{19}}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline & & & & -\frac{5\sqrt{\frac{95}{3}}}{7} & -\frac{\sqrt{95}}{7} & 0 & 0 & 0 \end{array}\right)^{\mathcal{D}}.$$

# System Analysis Properties

- Determination of the structural invariants and properties of descriptor systems
  - controllability, reachability and observability matrices
  - finite and infinite decoupling zeros
  - finite and infinite system poles and zeros
  - finite and infinite invariant zeros
  - finite and infinite transmission poles and zeros
  - Controllability, reachability, observability, detectability, stabilizability, stability tests.

# Descriptor State Space Models

Analysis of the descriptor state-space model of a simple RLC circuit. Zeros-Poles

- The **Smith McMillan form** of the pencil

```
McMillanDecomposition[s*e - a, s][[1]]//Factor
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{C L s^2 + C R s + 1}{C L} \end{pmatrix}$$

- The **Smith McMillan form** of the pencil at infinity

```
McMillanDecomposition[s*e - a, s, ForbiddenPolesArea
-> InfinityPoint][[1]]
```

$$\begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

*No zeros at infinity*

# Descriptor State Space Models

## Analysis of the descriptor state-space model of a simple RLC circuit. Zeros-Poles

- Infinite transmission poles-zeros (Infinite poles-zeros of $C(sE-A)^{-1}B$ )

```
tf=TransferFunction[dss];
McMillanDecomposition[tf[s], s, ForbiddenPolesArea ->
  InfinityPoint][[1]]
```

$$\left( \frac{1}{s^2} \right)$$

*One transmission zero at infinity of order 2*

- Infinite input-decoupling zeros (Infinite zeros of $\left[ (sE-A) \quad B \right]$ )

```
sc = AppendRows[s*e-a, b] ;
McMillanDecomposition[sc, s, ForbiddenPolesArea ->
  InfinityPoint][[1]]
```

$$\begin{pmatrix} s & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

*No decoupling zeros at infinity*

# Descriptor State Space Models

Analysis of the descriptor state-space model of a simple RLC circuit.
Controllability-Observability

**Consider the RLC circuit with R=L=C=1.**

```
dssrlc=dss/.{R->1, L->1, C->1};
```

```
Cm=ControllabilityMatrix[dssrlc]
Controllable[dssrlc]
```

$$\begin{pmatrix} -\frac{\sqrt{\frac{3}{95}}}{2} & -\sqrt{\frac{3}{95}} & 0 & 0 & 0 \\ \frac{\sqrt{\frac{5}{19}}}{2} & -\frac{2}{\sqrt{95}} & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{46}{105}} & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

True

# Time domain responses

- Symbolic approach (StateResponse and OutputResponse)
  - When supplied the input and the initial conditions, attempts to calculate the state and output response respectively.
- Simulation based approach (SimulationPlot)
  - Approximate numerical solutions.

# Time domain responses

Response of the descriptor state-space model of a simple RLC circuit.
State Response

```
dssrlc=dss/.{R->0.5, C->0.4, L->1};
x0 = {0,0,0,0};ut = {DiracDelta[t]};
xd=StateResponse[dssrlc,ut,t,InitialConditions->x0]//N
```

$$\{e^{-0.25\,t}\,(1.\cos(1.56125\,t) - 0.160128\sin(1.56125\,t))\,\theta(t),$$
$$1.\,\delta(t) + e^{-0.25\,t}\,(-0.5\cos(1.56125\,t) - 1.52122\sin(1.56125\,t))\,\theta(t),$$
$$1.60128\,e^{-0.25\,t}\sin(1.56125\,t)\,\theta(t),$$
$$e^{-0.25\,t}\,(0.5\cos(1.56125\,t) - 0.0800641\sin(1.56125\,t))\,\theta(t)\}$$

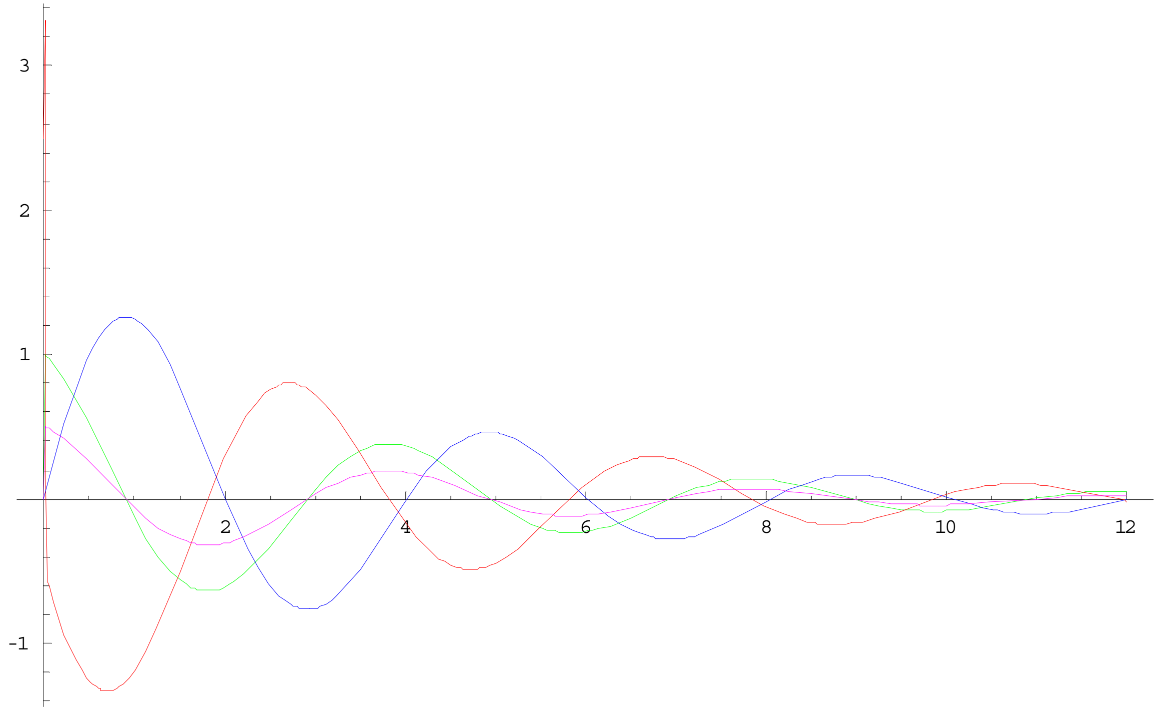
$\theta(t)$ is the unit step function

# Time domain responses

Response of the descriptor state-space model of a simple RLC circuit.

State Response

```
Plot[Evaluate[xd/.DiracDelta->Gaussian],{t,-0.01,12},
PlotStyle->{RGBColor[0,1,0],RGBColor[1,0,0],RGBColor[0,0,1],
RGBColor[1,0,1]},PlotRange->All]
```




$I$

$V_L$

$V_C$

$V_R$

# Design Synthesis Techniques

- Stabilizing compensator design, asymptotic tracking, model matching and disturbance rejection.
- Descriptor system interconnections such as series, parallel, feedback and generic interconnection.
- Pole assignment techniques

# Design Synthesis Techniques

## Pole assignment of a simple RLC circuit.

- Assign the poles of the system to {p1, p2} by **constant state feedback**

```
f=StateFeedbackGains[dss, {p1, p2},
      Method->FiniteDescriptorPoleAssignment]
```

$$\begin{pmatrix} -5L(\mathrm{p1}+\mathrm{p2})-9R & 6 & 5CL\,\mathrm{p1}\,\mathrm{p2}+1 & 10 \end{pmatrix}$$

McMillanDecomposition[s*e-(a+b.f), s][[1]]//Factor

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -(\mathrm{p1}-s)(s-\mathrm{p2}) \end{pmatrix}$$

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- Acknowledgements
  - Thanks to Wolfram Research and especially to Dr. Igor Bakshee for their interest and valuable help.

- Further development
  - Advanced Numerical methods for descriptor control systems.